Students' vizualisation of functions from secondary to tertiary level

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In this paper, we report a test which was proposed to students entering University (more than 500 students). The test was built to help teachers identify students' strength and weakness in some important mathematics topics, especially limits of functions. The test's analysis shows some specific abilities of students which surely can be used to introduce new knowledge involving the local perspective and formalism at the beginning of the university.

Key words: mathematics, functions, university students, activity, visualization, local perspective

In this article we want to investigate one problem that arises in the transition between secondary school and university concerning the concept of functions. We make an attempt to introduce specific students' activities with functions (called DWP), similar to those introduced by Duval (1999) about figure in the geometrical frame. The results may suggest that university teachers can built on these specific activities, to introduce some better students' understanding, involving the local perspective on functions, with its formalism and its relations to the other ones (global, point-wise).

NON-ICONIC VISUALIZATION AND DECONSTRUCTIONS WITH PERSPECTIVES OF FORMULAS

Rogalski (2008) and Vandebrouck (2011) have considered the notion of perspectives. In fact, different perspectives can be adopted concerning functions: a point-wise perspective – associated to function values in some particular points - a global perspective – ability to appreciate some global properties of functions such as variations, parity… - and a local perspective – ability to appreciate some local properties such as behavior near a point or near infinity.

The present paper aims to understand how students deal with perspectives on functions which are only given with their algebraic formulas. We examine the way perspectives can be useful when students have to compute some limits of functions given by their algebraic representations (formulas). The current practices of teaching in secondary schools in France don't give a qualitative vision of functions and reinforce tasks belonging to the algebraic frame (computations of limits with algebraic rules which are more or less demonstrated, of derivative...). These practices seem to erase the perspectives which can be adopted on these objects.

For our focus, we introduce the notion of *deconstruction with perspective* (DWP) of a formula in a similar way Duval (1999) has introduced the dimensional deconstruction of a figure in the geometrical setting. The dimensional deconstruction is a specific activity with geometrical figure linked to the ability to identify objects of dimensions 0, 1 or 2 in a whole complex figure (in 2 or 3 dimensions). In a similar way, the DWP is an activity which is specific of the analysis setting as we will explain below. This notion has been already introduced in Kuzniak and al (2015). It can also be applied for graphs of functions as Vivier does about tangents of curves, however without using this new terminology of deconstruction (Montoya Delgadillo & Vivier, 2015).

As the dimensional deconstruction does in the geometrical frame, the DWP supposes first of all a non-iconic visualization of the formula. We use the concept of visualization also introduced by Duval (1999) in the geometrical setting, but as Duval says, visualization can be produced in any register of representation. Duval distinguishes two type of visualizations: the iconic and the non-iconic. The latter involves some highlights, a global apprehension of the representation, may be a kind of classification, and some embarked properties.

We shall now give two examples of DWP, one about global DWP of algebraic formula and one about local DWP.

As it was focused in Vandebrouck's previous papers (Vandebrouck, 2011), only for experts formula can represent a function from a global perspective. For instance, the formula $x^2+\sqrt{x}+exp(x)$ represents a growing function on R^+. The non-iconic visualization of the formula by an expert allows him to identify three terms x^2, $\sqrt{x}$ and $exp(x)$, each term representing a growing function on R^+. This decomposition of the formula $x^2+\sqrt{x}+exp(x)$ into three growing functions can be named a *decomposition with global perspective*. For students, interpreting an algebraic formula as a function from a global perspective seems only natural for elementary functions *exp, ln*, x^2, $\sqrt{x}$, whose global properties – variations for instance - are well known. For more complex algebraic formulas, the most natural perspective is the point-wise one: non experts are only able to have an iconic visualization, using the formula as a dark box, associating $f(x)$ to x.

We notice that the *decomposition with global perspective* of a formula is more complex than identifying sums, products, quotients, several factors and so on, which is only a usual algebraic decomposition. For instance, the algebraic deconstruction of the formula $x^2+\sqrt{x}+exp(x)$ is the *sum* $\{x^2 / \sqrt{x} / exp(x)\}$. It is well done when we want to derivate or integrate the formula. We postulate that the algebraic decomposition doesn't suppose a non-iconic visualization, that is to say the non-iconic visualization is more complex. Many students are not able to visualize the function in a non-iconic way. To show this function is growing, they only identify the three algebraic terms of the sum – algebraic decomposition, iconic - and then compute the derivative as a sum, which is positive on R^+.

The second example (about local DWP) is about computing a limit of a function. For instance, the formula $(x^2+3x+1)/ln(x)$ represents a function on R^+. Let's compute the limit of the function at $+\infty$. As experts, we adopt a non-iconic visualization of the formula and we are able to operate a *decomposition with local perspective*. Near $+\infty$, the function is equivalent to $x^2/ln(x)$ – we must forget some negligible terms, a difficult activity. Moreover $ln(x)$ represents a negligible function compared with x^2. So the limit of the function represented by $(x^2+3x+1)/ln(x)$ is $+\infty$. Near 0^+, we do the same kind of local DWP. Of course, such decompositions have their limitations for students; as experts, we have some expert knowledge about sum, product and quotient of equivalent functions.

The issue in this paper concerns the students' ability to enter in such DWP after their algebraic practices at secondary school. That is to say we wonder in which way secondary teaching still allow students to develop such reasoning with DWP. If students are able to operate *decomposition with global and local perspective*, we suppose that they are more fluent with function in their formalism (local) at the beginning of the university: $f \sim g$, $f = o(g)$ and so on.

METHODOLOGY

In order to answer to this question, we had the opportunity to analyze answers of a diagnostic within the EVALAC[1] project at University Paris Diderot. All students entering university in scientific teaching were asked to answer an online questionnaire including 5 limits randomly selected among the 21 limits given in annex 1. 513 students answered the questionnaire, coming directly from secondary school. The limits were chosen among an IREM group by teachers from secondary schools and universities. Several issues about limit of functions were chosen such as algebraic classical rules were no sufficient to answer them.

Moreover, 6 students were interviewed while answering the questionnaire. All of them were students from Terminale S class in Lycees (grade 12, last course of the secondary school before a scientific baccalaureate). The focus of the questionnaire dealed with the cognitive way students answered questions about the limits. The question they had and the answers they gave are given in annex 2.

Statistics are given in annex 1. Not all students were from scientific baccalaureate classes so we can only take into account the highest or weakest percentages.

In order to analyze a priori the limits of functions, we draw on the task-analyzing tools (grills of complexity) proposed by Robert (1998): Do the limit calculations call

[1] http://www.ldar.univ-paris-diderot.fr/EVALAC

only for immediate applications of algebraic rules (direct substitution and algebra of limits), or, on the contrary, do they call for adaptations (especially for indeterminate form), sub-tasks (apply an algebraic rule to clear the indeterminate form for instance) and/or necessity for students to recognize other knowledge to be used (using DWP for instance)?

RESULTS AND EXAMPLES OF STUDENT RESPONSES

The first general observation is the rather poor rate of correct answers. This can be partly explained by the fact that not all students passed a science baccalaureate, even though they were highly predominant. This observation restricts the interpretations that can be made. Indeed some students who passed the test were not skilled enough with the theoretical knowledge about limits - such as the definition of a limit or the algebra' rules about limits.

The second general observation is the fact that the task-analyzing tool can't explain a lot of the results. Indeed, most of them are not significantly better when tasks are easier according to our a priori analysis. This observation is reinforced when we have identified that some algebraic rules which could be applied directly. The best success rates are on $(x^2-1)/(x+3)$ at $+\infty$ (indeterminate form, 81%) and on $1/(x+1)$ at $+\infty$ (that is not an indeterminate form, 85%). The limit of $(x-1)/(x+1)$ at $+\infty$ (indeterminate form) collects only 52% of correct answers even though students can directly apply a rule.

A last remark is the well-known difficulty linked to the idea that x is always positive. For instance, 22% of students think that the limit of $exp(-x)$ at $-\infty$ is 0.

Student's algebraic difficulties

The first six questions were about functions $exp(-x)$, $ln(1+x)$ et $ln(1/x)$. There basically are compositions of limits or substitutions. There is no decomposition to operate. The iconic visualizations can be sufficient enough because students can substitute and compose limits very algebraically. However, the results (between 50 and 70%) are not significantly higher than average.

The four next limits are about the functions $exp(x)-x$¸ $exp(-x)ln(x)$ and $exp(x)(1-\sqrt{x})$. Students can also manipulate algebra rules about limits. Here, we observe that the existence of an algebraic indeterminate form ($\infty-\infty$, $0 \times \infty$…) is not a criteria of difficulty. For instance, the limit of $exp(x)-x$ is better found at $+\infty$ (78%) than at $-\infty$ (55%) whereas only the first one is an indeterminate form $\infty-\infty$ which has to be cleared. In the same way, the limit of $exp(-x)ln(x)$ at $+\infty$ is better found (70%) than the limit of $exp(x)(1-\sqrt{x})$ at $+\infty$ (56%), whereas only the first one is an indeterminate form $0 \times \infty$.

The computation of the limit of $(exp(-x)-1)/(exp(-x)+1)$ at $-\infty$ is also interesting. Students have to identify a quotient. It is an algebraic deconstruction, traditional in algebraic activities, as we already said (students used to apply such deconstructions, especially for derivative computation). Then they have to identity that it is an indeterminate algebraic form belonging to the category ∞/∞. Students should then divide the numerator and the denominator by *exp*(-*x*), the dominating term. This factorization requires algebraic and functional knowledge. For instance 1/*exp*(-*x*)=*exp*(*x*). We observe that the percentages of answers are very scattered, with only 30% of good answers.

Concerning limits of rational functions at +∞, students know the algebraic rule of factorization which are traditional. However, as in the previous example, these kind of factorization are not so easy for them. The limit of *(x-1)/(x+1)* at +∞ is only succeeded with 52% of good answers. One can identify some effects of wrong rules application, for instance ∞ / ∞ = 0 (it surely justifies 20% of them answer 0) or ∞ / ∞ = ∞ (19% of them answer ∞). For instance, student B says for the function *(2x-2)/(x+1)* at + ∞ « *at the top, + infinite, at the bottom, + infinite, so + infinite, we have it in the array of indeterminate forms* ». So the algebra of limits seems not to be very well known. It can explain some big mistakes.

Students' ability to adopt some DWP – first kind of evidences

It seems that for expressions with the exponential function, such as *exp(x)-x* and *exp(-x)ln(x)* near + ∞, students are able to identify that the exponential function dominates. For instance, student B says « *exp is very powerful, very fast, if I replace x by a great number…* » and student F, who must have answered the limit of these two functions says about *exp(x)(1-√x)*: « *the square root of x goes to + infinite, so minus square root of x goes to – infinite, +1, it's again – infinite, the exponential grows faster than square of x so it wins, so + infinite* ». And about *exp(x)-x* the same student says: « *+ infinite, it's the same answer, exp(x) goes to + infinite, -x goes to – infinite but the exponential goes faster so + infinite* ». In fact, this student failed in the algebraic deconstruction: he didn't see that it was first a sum and then a product. But his main argument is based on the exponential domination.

For the limit of *exp(x)(1-√x)* at + ∞, student F doesn't take into account if the algebraic form is indeterminate or not. He doesn't see that it is not an indeterminate form and his knowledge about the exponential domination leads him to a mistake. In the general results, we really observe a great failure with this calculation of limit: 44% of the students answer + ∞ as the limit of *exp(x)(1-√x)* at + ∞. It seems that in all computation of a limit, the exponential function dominates. Moreover, as we mentioned earlier, students have difficulties with the signs of the quantities.

For the limit of *(exp(-x)-1)/(exp(-x)+1)* at - ∞, we can assume that many students do not have enough knowledge to operate the awaited algebraic manipulations. Some of them seem to operate a deconstruction with local perspective to visualize – in a non-iconic way - that the function behaves like 1 near + ∞. For instance, student C says « *exp(-x)-1 goes to + infinite, exp(-x)-1 goes to + infinite, so it goes to 1, -1 et +1 we* *can neglect them…*». Moreover, 26% of students answer -1, which can be explained by the traditional mistake of sign (x is always positive) and $exp(-\infty)=0$. 26% of students again answer 0, may be here again because they visualize that the exponential function dominates, even if it is a quotient, but it is a major hypothesis.

Student's amalgam between point wise and local perspective

Let's come back to the six first questions. Students may have an amount of skills that work both with substitution method (for continuous functions) and with real limits in a generalized algebra. It seems that functions are always continuous over [-∞ ; + ∞] with extended values such as $exp(-\infty)=0$; $exp(+\infty)=+\infty$; $exp(0)=1$; $ln(+\infty)=+\infty$; $ln(0)=-\infty$; $ln(1)=0$. It is an implicit extension of R to the extended real number line R barre. The interviewed students do not seem to distinguish between « it is » and « it tends » which are used indifferently ; as student A says about *ln(1+x)* « *it is a composed form ln(u(x)), 1/x goes to* 0^+*, it's a formula from the course, we put X=1/x, ln(X) it gives minus infinite when X tends to* 0^+ » and after « *it's the same, it's a composed form, X=-x, x goes to 0 so X goes to 0 and exp(0)=1* ». For this student, there is no distinction between a substitution or a numerical composition in a continuous function (point-wise perspective) and a real limit (local perspective).

This phenomena also appear when students seem to operate some DWP: for instance, student C says for the limit of *exp(-x)ln(x)* at + ∞ « *exponential at - infinite, it's 0, ln x goes to + infini, and 0 times infinite it's 0* ». We can call this phenomena double DWP – point wise and local. As another example, computing the limit of $sin(x)/x$ at 0, student C says « *sin(0) it's 0, and x it only goes to 0, so the limit is 0* ». He operates a point-wise decomposition of the numerator (as if x equal 0) and a local decomposition of the numerator (x goes to 0). Such reasoning can explain some qualitative wrong rules teachers of the IREM group have confirm, for instance « *0 over something equals 0* ». These wrong rules can justify that 35% of students answer 0 for the limit of $(x^2-1)/(x-1)$ at 1 and 30% answer 0 for the limit of $(2x-2)/(x+1)$ at 1.

In the same way, qualitative rules such as « *something over 0 gives infinite* » and « *something over infinite gives 0* » are also associated with juxtaposed point-wise and local decomposition. We can correlate these rules with strong rates of success when they are right and strong rates of failure when they are wrong. For instance, if we avoid students' traditional problems with signs, we can say that 82% of students (53%+29%) answer correctly an infinite (-∞ or + ∞) for the limit of $(x^2-1)/(x+3)$ at -3 and 60% of them (37% + 23%) answer correctly an infinite for the limit of $(2x-2)/(x+1)$ at -1^+. Student C says for $(2x-2)/(x+1)$ at -1^+ « *the more we divide by one 0, the greater it is. As it is negative (the numerator) it is minus infinite* ». The application of such rules can also explain that 31% of students answer +∞ for the limit of $(x^2-1)/(x-1)$ at 1^+, which is a wrong answer.

Students' ability to adopt some DWP – second kind of evidences

As we have already said, it seems that many students are unable to deal with the application of algebraic rules to clear indeterminate forms (for instance factorization by the dominant terms in a quotient or identification of a basic algebraic identity – remarkable - for the case $(x^2-1)/(x-1)$) at 1^+). However, some of these students seem more comfortable with DWP: for instance students are less successful in the calculation of the limit of $(x-1)/(x+1)$ at $+\infty$ (52% of good answers) than in the one of the limit of $(x^2-1)/(x+3)$ at $+\infty$ (81%). In the same way, student F says « *x^2 goes to infinite faster than x, so the limit is + infinite* ». Furthermore, 37% of the students answer $+\infty$ for the limit of $(2x-2)/(x+1)$ at $+\infty$, which is wrong. The origin of the mistake can be found by students' focusing on the qualitative argument $2x$ grow faster to $+\infty$ than x. Concerning student D, he doesn't succeed in applying an algebraic rule in a right way. However he still finds the right answer, stating « *it is twice more above than below* ». We can clearly say that he has operated a DWP instead of calculating with difficult algebraic rules.

SYNTHESE

The confrontation to the results of a real test over students is not so easy, especially when the framework of the test is not stable. For instance not all students have the same knowledge about limit. The conclusions of this paper have to be confirmed and refined. However these conclusions seem original. The notions of non-iconic visualization and decompositions with point-wise, global or local perspectives seem enough robust to explain and characterize specific students activities in the analysis setting.

Our paper suggests that students have algebraic abilities which are weak in order to compute limits: they have difficulties to identify the kind of indeterminate forms ($\infty-\infty$, $0 \times \infty$…). Indeed, there is no significant difference of results whether the algebraic form is determinate or not. Students also find difficulties with algebraic rules and algebraic manipulations to clear indeterminate forms.

Moreover, it seems that they amalgam point-wise and local perspectives, embedded in algebraic procedures. They have developed a specific knowledge about a generalized algebra ($exp(-\infty)=0$; $ln(0)=-\infty$…). In this specific mathematical area, local limit calculations and point-wise substitutions are mixed. In consequence, students are able to amalgam point-wise substitutions and decompositions with local perspective on the same formula.

However, a vicious circle may become a virtuous circle. Without a sufficient work involving the perspectives on functions – mostly by algebraic calculations, a lack of graphical tasks and coordination of the two registers, the internalization of few elementary functions… - students do not understand properly the technical rules they are asked to remember and apply. In particular, they are not able to identify which forms are indeterminate or not. Moreover, they have difficulties in algebraic calculus (for instance isolate commons factors in complex expressions). Consequently, it seems they may have developed qualitative knowledge which appear near to the DWP we have introduced above.

This conclusion helps to explain why the limit of *(x²-1)/(x+3)* at +∞ is better succeeded than the one of *(x-1)/(x+1)*. It helps to explain why many students find that the limit of *(2x-2)/(x+1)* at +∞ is +∞, which is wrong. It helps again to explain that near half of them answer + ∞ for the limit of *exp(x)(1-√x)* at + ∞, considering surely that the exponential function dominates and not operating the algebraic rule. This ability to operate such (sometime partial) global and local decompositions instead of algebraic operations, substitutes perhaps knowledge about the algebra of limits. This algebraic knowledge appears very technical for students, and it doesn't have any meaning for them – surely because it is not relied to perspectives.

Of course, this new kind of knowledge about DWP is not totally operational. There never is institutionalization about it during classroom. Students use these decompositions but with mistakes, without any mastery. This leads them to good answers as well as big mistakes. We also observe some decompositions with different perspectives on the same formula, with the automatic rules we have listed at the end of the previous paragraph. May be the teaching in secondary school could built on these new kind of knowledge instead of developing algebraic skills with less and less meaning for students?

ANNEX 1

exp(-x)	+ ∞	**0**	**70%**	**CA**
		+ ∞	11%	
		- ∞	11%	
		1	10%	
	- ∞	0	**22%**	
		+ ∞	**65%**	**CA**
		- ∞	7%	
		1	6%	
	0	0	16%	
		+ ∞	10%	
		- ∞	9%	
		1	**65%**	**CA**
ln(1+x)	+ ∞	0	19%	

(exp(-x)-1)/ (exp(-x)+1)	- ∞	**1**	**30%**	**CA**
		+ ∞	16%	
		0	**26%**	
		-1	**26%**	
1/(x+1)	+ ∞	**0**	**85%**	**CA**
		+ ∞	8%	
		1	7%	
(x-1)/(x+1)	+ ∞	-1	8%	
		1	**52%**	**CA**
		0	**20%**	
		+ ∞	19%	
	0	**-1**	**73%**	**CA**
		1	9%	
		0	14%	

		$+\infty$	**76%**	**CA**
		$-\infty$	5%	
	0^+	**0**	**68%**	**CA**
		$+\infty$	**23%**	
		$-\infty$	9%	
$ln(1/x)$	$+\infty$	0	**23%**	
		$+\infty$	12%	
		$-\infty$	**50%**	**CA**
		PDL	14%	
$exp(x)-x$	$-\infty$	0	**18%**	
		$+\infty$	**55%**	**CA**
		$-\infty$	**27%**	
	$+\infty$	0	11%	
		$+\infty$	**78%**	**CA**
		$-\infty$	11%	
$exp(-x)ln(x)$	$+\infty$	**0**	**70%**	**CA**
		$+\infty$	16%	
		$-\infty$	14%	
$exp(x)(1-\sqrt{x})$	$+\infty$	**$-\infty$**	**56%**	**CA**
		$+\infty$	**44%**	
$(exp(-x)-1)/(exp(-x)+1)$	$+\infty$	**-1**	**60%**	**CA**
		$+\infty$	8%	
		0	18%	
		1	13%	

NL means No limit - CA means Correct Answer Second column is the point where the limit is asked, third column is the four propositions.

		$+\infty$	3%	
$(x^2-1)/(x-1)$	1^+	**2**	**10%**	**CA**
		0	**35%**	
		1	**23%**	
		$+\infty$	**31%**	
$(x^2-1)/(x+3)$	$+\infty$	**$+\infty$**	**81%**	**CA**
		0	14%	
		-1/3	5%	
	-3^-	$+\infty$	**29%**	
		$-\infty$	**53%**	CA
		8	16%	
$(2x-2)/(x+1)$	$+\infty$	**2**	**56%**	**CA**
		$+\infty$	**37%**	
		-2	7%	
	-1^+	**$-\infty$**	**37%**	**CA**
		$+\infty$	**23%**	
		0	**30%**	
		-4	9%	
$sin(x)/x$	0+	0	**20%**	
		1	**20%**	**CA**
		NL	**32%**	
		$+\infty$	**26%**	
$sin(x)$	$+\infty$	**NL**	**51%**	**CA**
		$+\infty$	12%	
		1	**37%**	

ANNEX 2

Student A

$ln(1/x)$	$+\infty$	$-\infty$	CA
$exp(-x)$	0	1	CA
$sin(x)$	$+\infty$	NA	
$(exp(-x)-1)/$	$-\infty$	1	CA

Student D

$(x^2-1)/(x-1)$	1^+	2	CA
$(x^2-1)/(x+3)$	-3^-	$-\infty$	CA
$(2x-2)/(x+1)$	**$+\infty$**	**2**	**CA**
$exp(x)-x$	$-\infty$	$+\infty$	CA

(exp(-x)+1)			
No data			

(x-1)/(x+1)	+ ∞	**1**	**CA**

Student B

exp(-x)	- ∞	+ ∞	CA
(2x-2)/(x+1)	-1^+	- 4	
(2x-2)/(x+1)	+ ∞	+ ∞	
exp(-x)	0	1	CA
No data			

Student C

exp(-x)	- ∞	+ ∞	CA
(2x-2)/(x+1)	-1^+	- ∞	CA
sin(x)/x	0+	0	
exp(-x)ln(x)	+ ∞	**0**	**CA**
(exp(-x)-1)/ (exp(-x)+1)	+ ∞	**1**	**CA**

Student E

ln(1/x)	+ ∞	- ∞	CA
sin(x)	+ ∞	NL	CA
(x²-1)/(x+3)	+ ∞	+ ∞	CA
exp(x)-x	- ∞	+ ∞	CA
1/(x+1)	+ ∞	0	CA

Student F

sin(x)/x	0+	NL	
(exp(-x)-1)/ (exp(-x)+1)	- ∞	0	
exp(x)(1-√x)	+ ∞	+ ∞	
exp(x)-x	+ ∞	+ ∞	**CA**
(x²-1)/(x+3)	-3^-	- ∞	**CA**

REFERENCES

Duval, R. (1999) Representation, vision and visualization: Cognitive functions in mathematics thinking. Basic issues for learning. In F. Hitt y M. Santos (Eds.), *Proceedings of the 21st North American PME conference*, *1,* 3-26

Kuzniak, A., Montoya Delgadillo, E., Vandebrouck F., Vivier L. (2015) Le travail mathématique en analyse de la fin du secondaire au début du supérieur : identification et construction, *Actes de la 18ième école d'été de didactique des mathématiques,* to appear

Montoya Delgadillo, E., Vivier, L. (2015). ETM de la noción de tangente en un ámbito gráfico - Cambios de dominios y de puntos de vista, *Proceedings of CIAEM XIV*, 5-7 june 2015, Tuxtla Gutiérrez, Chiapas, México.

Robert, A. (1998). Outil d'analyse des contenus mathématiques à enseigner au lycée et à l'université. *Recherches en Didactique des Mathématiques*, *18* (2), 139-190.

Rogalski, M. (2008). Les rapports entre local et global : Mathématiques, rôle en physique élémentaire, questions didactiques. In L. Viennot (Ed.), *Didactique, épistémologie et histoire des sciences* (pp. 61-87). Paris: PUF.

Vandebrouck F. (2011) Students conceptions of functions at the transition between secondary school and university, *Proceedings of the Conference CERME 7*, Rzeszow, Poland, 9-13 février 2011